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ABSTRACT

The implementation of the program used in this study, TPIXIE, was predicated on the assumption that acquiring the skill to diagnose student errors was important for teacher trainees. The TPIXIE system (part of the PIXIE family of intelligent tutoring systems) was designed to present users with a series of task-student-answer pairs (where the student's answer was incorrect). The system presents several sets of tasks together with a student's responses, and then presents further tasks and asks the user to predict the responses. If the user is unable to do this, the cycle is repeated. The experiment reported compared the performance of two groups of teacher trainees, one using TPIXIE and the other a variant of PIXIE which merely asked the user to work algebraic tasks. The experiment suggested that the users benefited from interacting with TPIXIE, but transfer of skill was not demonstrated to analogous tasks. The paper concludes with suggestions for a further experiment. (TW)

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TPIXIE: A computer program to teach diagnosis of algebra errors:

An interim report

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TPIXIE: A computer program to teach diagnosis of algebra errors:

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The implementation of the program used in this study, TPIXIE, was predicated on the assumption that acquiring the skill to diagnose student errors was important for trainee teachers. The TPIXIE system (part of the PIXIE family of intelligent tutoring systems) was designed to present users with a series of task-student-answer pairs (where the student's answer was incorrect). The system presents several sets of tasks together with a student's responses, and then presents further tasks and asks the user to predict the responses. If the user is unable to do this, the complete student working is displayed, and the cycle is repeated.

The experiment reported here compared the performance of two groups of trainee teachers, one using TPIXIE and the other a variant of PIXIE which merely asked the user to work algebraic tasks. The experiment suggested that the users benefited preferentially from interacting with TPIXIE, but transfer of skill was not demonstrated to analogous tasks.

The paper concludes with suggestions for a further experiment.

this century. For example, Uhl studied errors in 1917 (Uhl, 1917), with subsequent work by researchers such as Buswell and Judd (1925), Brueckner (1930) and Buckingham (1933). This interest continues to the present (e.g., Bunderson & Olsen, 1983). For a list of selected references, see Ashlock (1982).

In addition to studying mathematical errors for their own sakes, some researchers have stressed the need to alert teachers to them for the sake of improving instruction (e.g., Cox, 1975; Fowler, 1980; Swan, 1983), since some students are reported to have confidence in their faulty procedures (Feghali, 1976; MacKay, 1975). According to West (1971), "There is hardly a skill in the teachers repertoire that is more important than the ability to identify pupil errors and to prescribe appropriate remedial procedures," and errors may even be "springboards" for students to understand mathematics (Borasi, 1986). Further, as Brown and Burton (1978) note, ignoring or misinterpreting students' errors may be detrimental to students' motivation:

> When a student's bug (which may only manifest itself occasionally) is not recognised by the teacher, the teacher explains the errant behaviour as carelessness, laziness, or worse, thereby often mistakenly lowering his opinions of the student's capbilities... From the student's viewpoint, the situation is much worse. He is following what he believes to be the correct algorithm and, seemingly at random, gets marked wrong (p. 285, italics in the original).

Training teachers to diagnose errors. Such concern has lead to efforts to train teachers to diagnose errors. Brown and Burton (1978) used a computer to tutor diagnosis of errors in subtraction successfully.

Since then, at least two further attempts at training teachers to diagnose bugs using computers have been made, one for addition and subtraction (De Corte, Verschaffel & Schrooten, 1986) and one for algebra (Schneider, Kelly, Blando, Martinak, Sleeman & Snow, 1986). [For an analogous study not using a computer tutor see Dodd, Jones and Lamb, (1975).]

De Corte et al., (1986) found that students who worked with their computer program (based on VanLehn's "Buggy Game") were superior to those in a control group on the ability to hypothesize a particular bug, and verify it by predicting the wrong answer that would be obtained for a set of tasks if that bug were to be used.

In a pilot study, Schneider et al., (1986) found that teachers were better at diagnosing algebra errors having worked with TPIXIE, part of a larger intelligent tutoring system (Sleeman, 1986). Although the teachers in the study enjoyed working with TPIXIE, their major criticism was that it did not present challenging tasks soon enough; this criticism that has since been addressed. This paper reports on a follow-on study, the purpose of which is to test the effectiveness of the revised TPIXIE, and suggest further improvements to it.

Transfer of Training. A common concern for training is how well it transfers to related and previously unencountered tasks. De Corte et al., (1986) did not find transfer of training with their program. The results on transfer from the TPIXIE pilot study (Schneider, et al., 1986) were encouraging, but not definitive. Therefore, transfer of training received further attention in this study.

Method

Subjects. Thirty-six elementary-school first-year teacher-trainees from a Scottish college of education served as subjects. Each student had completed a secondary-school course, and would have had training in mathematics including algebra. (Ideally, we would have liked to use secondary school teacher-trainees, but such students in the College had already been trained in diagnosis of errors.) Students were paid a nominal fee for their participation.

Materials. Two computer programs were used:

TPIXIE. TPIXIE (Sleeman, 1986) is designed to help the user diagnose a common bug between a set of equations (see Figure A). The user is shown a set of three task-student-answer pairs; or task-answer pairs for short, from which it is hoped that the user will determine the mal-rule the particular student is making. To test this, the user is presented with three further tasks to which they must respond by giving the response they believe corresponds to the student's buggy rule. If the user diagnoses the bug correctly, a new set of equations is presented. Otherwise, the target set of equations for that task level is reshown. If the user is unable to diagnose the common bug in the target equations, a facility exists to show the pupil's faulty working (see Figure B). Following such feedback, TPIXIE proceeds to the next task level. As the user progresses through the 16 sets of equations, the bugs generally become more complicated. TPIXIE records each response made by the user.

FPIXIE. FPIXIE presents a series of algebra equations one at a time and asks the user to solve them (see Figure C). FPIXIE comments simply on whether the answer was correct or not, and then presents a new item. In

general, each item is more difficult than the prior one. By using FPIXIE we were able to control for time spent on the computer, and for the domain area covered. FPIXIE is a part of the more general RPIXIE intelligent tutoring system (Sleeman, 1987).

Pretest. Subjects saw a test comprised of 28 task-answer pairs. No intermediate steps in the solution were shown; this allowed the investigators to assess diagnostic ability under the stingent conditions of limited information.

The items were arranged in sets: Sets 1, 2, 3, 4, and 6 had five items each; Set 5 had three items. Sets 1, 2, 4, 5 and 6 had a common faulty procedure (bug) underlying each error in their respective sets, whereas each of the task-answer pairs in Set 3 had a different bug. Set 3 was included to discourage subjects from presuming that the diagnosis of the first task-answer pair held for all others in the set.

Posttest.

The posttest was similar to the pretest in format. The same bugs were used to generate the items, except in the case of set 5, in which the square root of the final answer was taken. The square root was inadvertently omitted from the items in set 5 on the pretest, and so the bug underlying set 5 in the posttest was therefore more complex than that for set 5 on the pretest. The bugs in both pre- and posttests were based on previously observed students' protocols.

Unlike the other sets, sets 2 and 4 contained bugs not seen on TPIXIE. These sets were included to test for transfer of training.

Procedure

A pretest was administered to all the trainee-teachers taking part in the experiment; the group was randomly assigned to one of the two conditions. Over a period of seven days immediately following the pretest, they worked either the TPIXIE or FPIXIE programs for a single period of 50 minutes. Six days after the last teacher-trainees worked with the computer the entire sample was given the posttest. Teacher-trainees were allowed 50 minutes for both the pretest and posttest.

Results

Two teacher-trainees from the treatment condition (TPIXIE) were absent from the pretest and did not take part in the experiment. Their absence was unrelated to the experimental conditions. A further student was very poor at algebra and apparently did not understand what was required of her; consequently, her scores were dropped from the analyses. This left 16 students in the treatment condition, and 17 in the control (FPIXIE) condition.

Diagnosing error patterns. We wished to see how well the teacher-trainees, by condition, diagnosed the bugs underlying the sets of items that contain a single common bug (Sets 1, 2, 4, 5 and 6). Set 3 was omitted from both these analysis because there is no bug common to its five items. We performed the following analyses:

Majority Match. Credit is allowed for items in a set only if the majority of the task-answer pairs are matched to the known bug: so for sets 1, 2, 4 and 6 it is possible to score 0 or between 3 and 5 (as there are 5 items in these sets). Similarly, for set 5 which contains 3 items, scores can be 0 or in the range 2 to 3. The maximum total score possible is 23. Using this criterion, the TPIXIE condition outscored the control condition at a statistically significant level (TPIXIE M = 13.38, control M = 8.65, t (31) = 2.26, $p < .031$).

Full Match. Credit is allowed for a set only if all of the task-answer pairs are matched to the known bug: so scores could range from 0 to 5 (there being 5 sets in all). Using this criterion, the TPIXIE condition again outscored the control condition, but the difference was not statistically significant. (TPIXIE M = 2.25, control M = 1.47; t (31) =

1.54, $p < .14$)

Pre- To Posttest Gain. Both groups gained significantly from pretest to posttest on both match scores (see Tables 1 and 2).

Transfer of training. To test for transfer of training, neither the bug in set 2 nor that in set 4 was shown on TPIXIE. A comparison of the combined Majority Match scores on these sets showed that the TPIXIE group scored significantly higher than the Control group, (TPIXIE M = 6.75, Control M = 2.65, t (31) = 1.29, $p < 0.009$). The Full Match analysis showed no significant differences, however (TPIXIE M = .75, FPIXIE M = .41, t (31) = 1.39, $p < .17$).

Reliability Check. In scoring the raw data it was sometimes unclear as to whether or not an error had been diagnosed correctly. Some students provided explanations of errors which could have been interpreted as correct diagnosis, but which contained evidence leading the scorer to doubt this. Eight test sessions (four from the prestest and four from the posttest, each with two from the TPIXIE and two from the FPIXIE groups, but otherwise selected randomly) were rechecked by an independent scorer who was in 94.2% agreement with the original scorer. This casts some doubt upon the significance figures quoted in the rest of the paper. However, there was least grounds for doubt when a trainee teacher had consistently diagnosed a full set of items. Thus the methods by which the analysis of the raw scores was performed, using multiple matches within sets (either majority match or full match) rather than item by item should have minimised any sources of error due to inconsistencies in the scoring.

Set 3. As mentioned in the Materials section, set 3 was included to check if subjects were developing a response set in which they merely diagnosed a bug for the first item in the set and then applied it to the others. This appeared not to be the case. Out of a possible score of 5 on this set, the TPIXIE mean was 4.00, and the control mean was 3.47, indicating that each item was being considered on its own merits. There was no significant difference between these means: t (31) = 1.41, p < .17. [We wish to make no further claims regarding this result, because on set 3 on the pretest, for reasons we cannot explain, the TPIXIE mean (3.38) was significantly higher than the FPIXIE, mean (2.29), t (31) = 2.57, p < .015. It is important to note, however, that this anomaly did not occur for any of the other sets, and that the scores from set 3 did not enter into any of the other analyses in this paper.]

Discussion

The results of this study are generally encouraging for the further development TPIXIE. If the majority scoring criterion is used, the students in the TPIXIE condition score significantly better than those in the control group, and even when the more stringent criterion, a full match, is applied, its results favour TPIXIE. (The Type I error rate associated with this comparison is probably acceptable for a program under development.) These results, together with the finding that both groups improved significantly over their own pretest performances, leads us to conclude from these analyses that while students can learn diagnosis of errors without the aid of TPIXIE, those who work with TPIXIE are likely to be more effective diagnosticians.

At least two strong qualifications need to be made in this assessment of TPIXIE, namely, a) the subjects did not exactly match those for whom the program was intended (the subjects were elementary as opposed to secondary teacher-trainees), and b) the dependent measure was not as sensitive as we would have wished. For this sample only sets 2 and 6 seem to be discriminating between the groups; set 1 appears to be too simple, and sets 4 and 5 appear to be too difficult (see Table 3).

Both conditions saw the bugs for sets 2 and 4 on the pretest. These same bugs were not shown on TPIXIE. Nevertheless, teacher-trainees who worked with TPIXIE diagnosed these bugs on the posttest on a greater number of task-answer pairs than those in the control condition. In the next phase of the development of TPIXIE, we plan to include on the posttest bugs that will appear on neither the pretest nor in TPIXIE. In addition, we plan to include on the posttest, bugs similar to those on TPIXIE (to measure near transfer), and ones quite dissimilar (to measure

for transfer).

Comments by students. The students enjoyed using TPIXIE. Typical comments indicated, 1) "No teacher of diagnosis was needed", 2) "I had little difficulty working the program", and 3) "I liked the remedial option" (the one that explains the common bug if the user cannot discern it). All but two of the teacher-trainees drew domain-independent lessons from interaction with TPIXIE, such as the importance of making sure a pupil understands the rules of mathematics; the importance of having empathy for the learner who finds mathematics difficult; and the importance of knowing where a learner is going wrong in working tasks. The students who did not find TPIXIE helpful explained that they were elementary school teacher-trainees and found the domain subject (algebra) unrelated to their own work.

A number of suggestions for improvement of TPIXIE were given by these students, including:

1. The user should be allowed to return to the current set of task-answer pairs after having seen just a small number of lines of remedial explanation (at present the user is shown the entire misworking of the task). This number should be under control of the user. Such an option would allow the user to get "clues" as to the pupil's bug, which could then be used in a new attempt at solving the target task-answer pairs.

2. The comments used to encourage the user (see Figure A) should be varied, as they may become repetitious over the 50-minute session.

3. A variant of TPIXIE should be built for high-school students to help them diagnose algebra errors with the aim of improving their

performance at algebra.

Future work. Possible changes to the present system include:

1. Replacing the present algebra bugs with bugs that we now know (Martinak et al, 1987) are more common among high school students than those originally used. This change would make the skills learned more relevant to teachers.

2. As far as possible, TPIXIE should be tested on a sample of users that represents its target population; namely, trainee secondary school mathematics teachers.

3. Items should be pilot-tested to find ones that are neither too difficult nor too easy for the population under study; although introductory, easy items should be included on the tests and on TPIXIE for motivational purposes.

4. The number of items per set on the tests should be reduced to three in all cases, which would allow additional sets of tasks to be worked in the same amount of time.

5. Items should be selected so as not to be capable of being explained by more than one different common bug.

6. A TPIXIE variant might be developed that does not rely so heavily on the user's ability to abstract an error from a set of incorrectly worked tasks. Such a variant might, for example, first give the user a list of known pupil bugs. TPIXIE would then show task-answer pairs, and ask the user to diagnose the bugs. By comparing the results of these two

versions of TPIXIE we could begin to learn how important the task of having to discover the bug(s) is for subsequent diagnostic accuracy.

7. Finally, acting on the finding that those in the control group improved from pre- to posttest, one might consider including a pretest-posttest-only condition to see if gains similar to those of the control group in this study are made.

Table 1

Composite Majority Match Pre- and Posttest Mean Scores by Condition

	Descriptive statistics		Paired t-test		
Condition	Mean	SD	t	df	p
TPIXIE					
Pre	3.94	3.64	5.89	15	.000
Post	13.38	6.22			
Control					
Pre	3.12	3.92	5.05	16	.000
Post	8.65	5.79			

Note: Maximum score = 23

Table 2

Composite Full Match Pre- and Posttest Mean Scores by Condition

	Descriptive statistics		Paired t-test		
Condition	Mean	SD	t	df	p
TPIXIE					
Pre	0.31	0.60	4.58	15	.000
Post	2.25	1.53			
Control					
Pre	0.18	0.53	4.83	16	.000
Post	1.47	1.37			

Note: Maximum score = 5

Table 3

Posttest Descriptive Statistics by Item Set by Condition

	Set*				
Condition	1	2	4	5	6
TPIXIE **					
M	4.75	3.44	1.31	1.75	2.12
SD	0.68	2.22	2.06	1.29	2.28
Control ***					
M	3.94	1.59	1.06	1.29	0.94
SD	1.92	2.32	1.98	1.31	2.02

* Maximum score for Sets 1, 2, 4 and 6 = 5
Maximum score for Set 5 = 3. Set 3 was omitted from the analysis; see text.
** N = 16
*** N = 17

Figure A

TPIXIE

What is student #1 doing wrong ?
3 problems will be shown

The problem is 3x = 9
The student's answer was x =

The problem is 4x = 16
The student's answer was x =

The problem is 5x = 15
The student's answer was x =

Now solve the following problems like the student.
Take as much time as you need.

Solve (like the student) 4x = 24
Type Q to QUIT

x = 1/6

That looks OK. You seem to be on the right track.
Let's try another one.
Take as much time as you need.

Solve (like the student) 5x = 10
Type Q to QUIT

x = 1/2

That looks fine. I think you've got it!
Let's try one final item to be sure.
Take as much time as you need.

Solve (like the student) 2x = 8

x = 1/4

Bravo!
Do you wish to continue?

Figure B

TPIXIE: The remedial procedure at work. (Abbreviated format)
What is student #18 doing wrong?

17x = 2 * (4x + 3)

x =

8x = 2 * (3x + 5)

x =

19x = 2 * (4x + 3)

x =

Solve (like the student) 9x = 2 * (3x + 3)
Type Q to QUIT

Q

Now, let's take a look at how the student would have solved the problem.

The student would have solved the problem like this:

9x = 2 * (3x + 3)
9x = 6x + 3

Did not multiply second term in brackets. (Hit C and RETURN: C)

9x - 6x = 3

The x-term moved from the right-hand side to the left-hand side and the sign changed. (Hit C and RETURN: C)

3x = 3

Added or subtracted 2 x-terms.

x = (Hit C and RETURN: C)
Both sides divided by the coefficient.

x = 1

Numerator and denominator divided by greatest common factor.

Finished.

Figure C

FPIXIE

```
This is Task-Set 2
You will be asked to work at most 6 tasks at this level.

Solve  (f = Finished,  q = Quit early)
3 x = -9 x = - 3
f

Thank you.
You got that one right!
Well done

Do you wish to continue? Please type YES or NO:
y

Solve  (f = Finished,  q = Quit early)
4 x = -16
x = 4
f

Thank you,
but you didn't get that one right.
Do you wish to continue?
Please type YES or NO:
y

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TPIXIE: A computer program to teach diagnosis of algebra errors
A. E. Kelly, D. Sleeman, R. D. Ward & R. Martinak

a) p3 at top insert ABSTRACT

b) replace pages 18-20 with the attached

Figure A

TPIXIE

What is student #1 doing wrong ?
3 problems will be shown

The problem is 3x = 9
The student's answer was x = 1/3

The problem is 4x = 16
The student's answer was x = 1/4

The problem is 5x = 15
The student's answer was x = 1/3

Now solve the following problems like the student.
Take as much time as you need.

Solve (like the student) 4x = 24
Type Q to QUIT

S: x = 1/6

That looks OK. You seem to be on the right track.
Let's try another one.
Take as much time as you need.

Solve (like the student) 5x = 10
Type Q to QUIT

S: x = 1/2

That looks fine. I think you've got it!
Let's try one final item to be sure.
Take as much time as you need.

Solve (like the student) 2x = 8

S: x = 1/4

Bravo!
Do you wish to continue?

TPIXIE: The remedial procedure at work. (Abbreviated format)
What is student #18 doing wrong?

17x = 2 * (4x + 3)

x = 1/3

8x = 2 * (3x + 5)

x = 5/2

19x = 2 * (4x + 3)

x = 3/11

Solve (like the student) 9x = 2 * (3x + 3)
Type Q to QUIT

S: Q

Now, let's take a look at how the student would have solved the problem.

The student would have solved the problem like this:

9x = 2 * (3x + 3)
9x = 6x + 3

Did not multiply second term in brackets. (Hit C and RETURN: C)

9x - 6x = 3

The x-term moved from the right-hand side to the left-hand side and the sign changed. (Hit C and RETURN: C)

3x = 3

Added or subtracted 2 x-terms.

x = (Hit C and RETURN: C)
Both sides divided by the coefficient.

x = 1

Numerator and denominator divided by greatest common factor.

Finished.

Figure C

FPIXIE

```
   This is Task-Set 2
   You will be asked to work at most 6 tasks at this level.

   Solve  (f = Finished,  q = Quit early)
   3 x = -9
S: x = - 3
S: f

   Thank you.
   You got that one right!
   Well done

   Do you wish to continue? Please type YES or NO:
S: y

   Solve  (f = Finished,  q = Quit early)
   4 x = -16
S: x = 4
S: f

   Thank you,
   but you didn't get that one right.
   Do you wish to continue?
   Please type YES or NO:
S: y

 ......
```